

# PHOTON - AXION CONVERSION CROSS SECTIONS IN A RESONANT CAVITY

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## Abstract

Photon - axion conversions in the resonant cavity with the lowest mode are considered in detail by the Feynman diagram method. The differential cross sections are presented and numerical evaluations are given. It is shown that there is a resonant conversion for the considered process, in which the conversion cross sections are much larger than those of the wave guide in the same conditions. Some estimates for experimental conditions are given from our results.

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## 1. Introduction

The most attractive candidate for solution of the strong-CP problem is Peccei and Quinn mechanism, where the CP-violating phase  $\theta$  is explained by the existence of a new pseudo-scalar field, called the axion [1, 2, 3]. At present, the axion mass is constrained by laboratory searches [4, 5] and by astrophysical and cosmological considerations [6, 7] to between  $10^{-6}eV$  and  $10^{-3}eV$ . If the axion has a mass near the low limit of order  $10^{-5}eV$ , it is good candidate for the dark matter of the universe. Besides that, an axino (the fermionic partner of the axion) naturally appears in SUSY models [8, 9], which acquire a mass from three - loop Feynman diagrams in a typical range between a few eV up to a maximum of 1 keV [10, 11]. The candidates for dark matter can be appeared in different models. They were done in multi - Higgs extension of the standard model, in the 3-3-1 models [12, 13, 14] or in the supersymmetric and superstring theories [15].

Neutral pions, gravitons, hypothetical axions, or other particles with a two-photon interaction can transform into photons in external electric or magnetic fields, an effect first discussed by Primakoff [16]. This effect is the basis of Sikivie's methods for the detection of axions in a resonant cavity [17]. He suggested that this method can be used to detect the hypothetical galactic axion flux that would exist if axions were the dark matter of the Universe. Various terrestrial experiments to detect invisible axions by making use of their coupling to photons have been proposed [18, 19, 20], and results of such experiments appeared recently [21, 22, 23]. The experiment CAST [24] at CERN searches for axions from the sun or other sources in the universe. The experiment uses a large magnet from LHC and searches for the conversion of axions into photons. The potential of the CAST experiment for exotic particles was discussed [25]. The purpose of this paper is to consider conversions of photons into axions in the periodic EM field of the resonant cavity. We show that there is a resonant conversion for the considered process at the low energies. The text is organized as follows. In Sec.II we give the matrix element of the considered process. The differential cross section (DCS) for conversions in the resonant cavity with the  $TM_{110}$  mode is presented in Sec.III. Some estimates for experimental conditions are given in the last section - Sec. VI.

## 2. Matrix element

The axion mass and its couplings to ordinary particles are all inversely proportional to the magnitude  $v$  of the vacuum expectation value that spontaneously breaks the  $U_{PQ}(1)$  quasisymmetry which was postulated by Peccei and Quinn and of which the axion is the pseudo-Nambu - Goldstone boson. For the axion- photon system a suitable Lagrangian density is given by [6, 8]

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + g_\gamma \frac{\alpha}{4\pi} \frac{\phi_a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2} \partial_\mu \phi_a \partial^\mu \phi_a - \frac{1}{2} m_a^2 \phi_a^2 [1 + \mathcal{O}(\phi_a^2/v^2)] \quad (1)$$

where  $\phi_a$  is the axion field,  $m_a$  is its mass,  $\tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$ , and  $f_a$  is the axion decay constant and is defined in terms of the axion mass  $m_a$  by [9,12]:  $f_a = f_\pi m_\pi \sqrt{m_u m_d} [m_a(m_u + m_d)]^{-1}$ . Interaction of axions to the photons arises from the triangle loop diagram, in which two vertices are interactions of the photon to electrically charged fermion and an another vertex is coupling of the axion with fermion. This coupling is model dependent and is given by:  $g_\gamma = \frac{1}{2} \left( \frac{N_e}{N} - \frac{5}{3} - \frac{m_d - m_u}{m_d + m_u} \right)$  where  $N = \text{Tr}(Q_{PQ} Q_{color}^2)$  and  $N_e = Q_{PQ} Q_{em}^2$ . Tr represents the sum over all left - handed Weyl fermions.  $Q_{PQ}$ ,  $Q_{em}$ , and  $Q_{color}$  are respectively the Peccei- Quinn charge, the electric charge, and one of the generators of  $SU(3)_c$ . In particular in the Dine- Fischler - Srednicki - Zhitnitskii model [26, 27]:  $g_\gamma(\text{DFSZ}) \simeq 0.36$ , and in the Kim- Shifman- Vainshtein- Zakharov model [28, 29] ( where the axions do not couple to light quarks and leptons) :  $g_\gamma(\text{KSVZ}) \simeq -0.97$ .

Consider the conversion of the photon  $\gamma$  with momentum  $q$  into the axion  $a$  with momentum  $p$  in an external electromagnetic field. For the above mentioned process, the relevant coupling is the second term in (1). Using the Feynman rules we get the following expression for the matrix element [19]

$$\langle p | \mathcal{M} | q \rangle = -\frac{g_{a\gamma}}{2(2\pi)^2 \sqrt{q_0 p_0}} \varepsilon_\mu(\vec{q}, \sigma) \varepsilon^{\mu\nu\alpha\beta} q_\nu \int_V e^{i\vec{k}\vec{r}} F_{\alpha\beta}^{class} d\vec{r} \quad (2)$$

where  $\vec{k} \equiv \vec{q} - \vec{p}$  is the momentum transfer to the EM field,  $g_{a\gamma} \equiv g_\gamma \frac{\alpha}{\pi f_a} = g_\gamma \alpha m_a (m_u + m_d) (\pi f_\pi m_\pi \sqrt{m_u m_d})^{-1}$  and  $\varepsilon^\mu(\vec{q}, \sigma)$  represents the polarization vector of the photon.

Expression (2) is valid for an arbitrary external EM field. In the following we shall use it for the case, namely conversions in the periodic EM field of the resonant cavity.

Here we use the following notations:  $q \equiv |\vec{q}|, p \equiv |\vec{p}| = (p_o^2 - m_a^2)^{1/2}$  and  $\theta$  is the angle between  $\vec{p}$  and  $\vec{q}$ .

### 3. Conversions in a resonant cavity

For the sake of simplicity we choose the lowest nontrivial solution of the resonant cavity, namely the  $TM_{110}$  mode [30, 31]

$$\begin{aligned} E_z &= E_o \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right), \\ H_x &= \frac{i\varepsilon\pi}{\omega b} E_o \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right), \\ H_y &= -\frac{i\varepsilon\pi}{\omega a} E_o \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right), \end{aligned} \quad (3)$$

here the propagation of the EM wave is in the  $z$  - direction. Note that there is no E-wave of types (000), (001), (010), (100), (101), and (011).

Substitution of (3) to (2) gives us the following expression for the matrix element

$$\langle p | \mathcal{M} | q \rangle = \frac{g_{a\gamma}}{(2\pi)^2 \sqrt{p_0 q_0}} [(\varepsilon_1(\vec{q}, \tau) q_2 - \varepsilon_2(\vec{q}, \tau) q_1) F_z + \varepsilon_1(\vec{q}, \tau) q_0 F_x + \varepsilon_2(\vec{q}, \tau) q_0 F_y], \quad (4)$$

where  $p_0 \equiv q_0 + \omega$ , and

$$\begin{aligned} F_z &= -\frac{8E_o(q_x - p_x)(q_y - p_y) \cos[\frac{1}{2}a(q_x - p_x)] \cos[\frac{1}{2}b(q_y - p_y)] \sin[\frac{1}{2}d(q_z - p_z)]}{[(q_x - p_x)^2 - \frac{\pi^2}{a^2}][(q_y - p_y)^2 - \frac{\pi^2}{b^2}](q_z - p_z)}, \\ F_x &= -\frac{\varepsilon\pi^2 F_z}{\omega b^2(q_y - p_y)}, \\ F_y &= \frac{\varepsilon\pi^2 F_z}{\omega a^2(q_x - p_x)}, \end{aligned} \quad (5)$$

where  $a$ ,  $b$ , and  $d$  are three dimensions of the cavity and  $\omega$  is the frequency of the EM field. Substituting Eq.(5) into Eq.(4) we finally obtain the DCS for conversions

$$\begin{aligned} \frac{d\sigma(\gamma \rightarrow a)}{d\Omega} &= \frac{g_{a\gamma}^2 p_o}{2(2\pi)^2 q_0} \left[ (q_x^2 + q_y^2) F_z^2 + \left(1 - \frac{q_x^2}{q^2}\right) q_0^2 F_x^2 + \left(1 - \frac{q_y^2}{q^2}\right) q_0^2 F_y^2 \right. \\ &\quad \left. - 2q_x q_y F_x F_y - 2q_0 q_x F_y F_z + 2q_0 q_y F_x F_z \right]. \end{aligned} \quad (6)$$

Assuming that the momentum of the photon is parallel to the  $z$  - axis (the direction of the EM field) then Eq. (6) gets the final form

$$\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} = \frac{8\pi^2 g_{a\gamma}^2 E_o^2 q^2}{\omega^2} \left(1 + \frac{\omega}{q}\right) \left[ \frac{(p \sin \theta \cos \varphi')^2}{b^4} + \frac{(p \sin \theta \sin \varphi')^2}{a^4} \right]$$

$$\times \left[ \frac{\cos \frac{a}{2}(p \sin \theta \cos \varphi') \cos \frac{b}{2}(p \sin \theta \sin \varphi') \sin \frac{d}{2}(q - p \cos \theta)}{[(p \sin \theta \cos \varphi')^2 - \frac{\pi^2}{a^2}][(p \sin \theta \sin \varphi')^2 - \frac{\pi^2}{b^2}](q - p \cos \theta)} \right]^2, \quad (7)$$

where  $\varphi'$  is the angle between the x - axis and the projection of  $\vec{p}$  on the xy - plane and  $d\Omega' = d\varphi' d\cos \theta$ . From (7) it is easy to show that there are no conversions when  $\theta = 0$  and  $\varphi' = \frac{\pi}{2}$  and

$$\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} = \frac{8g_{a\gamma}^2 E_0^2 p^2}{\omega^2 \pi^2 \left(p^2 - \frac{\pi^2}{a^2}\right)^2} \left(1 + \frac{\omega}{q}\right) \cos^2\left(\frac{pa}{2}\right) \sin^2\left(\frac{qd}{2}\right), \quad (8)$$

for  $\theta = \frac{\pi}{2}$  and  $\varphi' = 0$ .

In the limit  $q \rightarrow \frac{\pi}{d}$ , Eq.(8) becomes

$$\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} = \frac{8g_{a\gamma}^2 E_0^2 p^2}{\omega^2 \pi^2 \left(p^2 - \frac{\pi^2}{a^2}\right)^2} \left(1 + \frac{\omega d}{\pi}\right) \cos^2\left(\frac{pa}{2}\right). \quad (9)$$

Next, if the momentum of the photon is perpendicular to the direction of the EM field, i.e., in the y - axis then from Eq. (6) we have

$$\begin{aligned} \frac{d\sigma(\gamma \rightarrow a)}{d\Omega''} &= \frac{32g_{a\gamma}^2 E_0^2 q^2}{(2\pi)^2} \left(1 + \frac{\omega}{q}\right) \left[ \left(q - p \cos \theta\right) - \frac{\pi^2}{\omega b^2} \right]^2 \tan^2 \varphi'' \\ &\times \left[ \frac{\cos \frac{a}{2}(p \sin \theta \sin \varphi'') \cos \frac{b}{2}(q - p \cos \theta) \sin \frac{d}{2}p(\sin \theta \cos \varphi'')}{[(p \sin \theta \sin \varphi'')^2 - \frac{\pi^2}{a^2}][(q - p \cos \theta)^2 - \frac{\pi^2}{b^2}]} \right]^2, \end{aligned} \quad (10)$$

where  $\varphi''$  is the angle between the z - axis and the projection of  $\vec{p}$  on the xz - plane.

From (10) we have  $\frac{d\sigma(\gamma \rightarrow a)}{d\Omega''} = 0$  for  $\theta = \varphi'' = 0$  and

$$\frac{d\sigma(\gamma \rightarrow a)}{d\Omega''} = \frac{g_{a\gamma}^2 E_0^2 a^2 d^2 q^2 \left(\omega q - \frac{\pi^2}{b^2}\right)^2}{2(2\pi)^2 \omega^2 \left(q^2 - \frac{\pi^2}{b^2}\right)^2} \left(1 + \frac{\omega}{q}\right) \cos^2\left(\frac{qb}{2}\right), \quad (11)$$

for  $\theta = \varphi'' = \frac{\pi}{2}$ , and in the limit  $p \rightarrow \frac{\pi}{a}$ .

From Eq.(11) we can see that this is the best case for the conversions. In this case, the DCS depends quadratically on the amplitude  $E_0$ , photon momentum  $q$  and two dimensions of the resonant cavity.

#### 4. Conclusion

The following consequences may be obtained from our results:

i) To compare the results with the wave guide( for details, see Ref. [20]) we can introduce

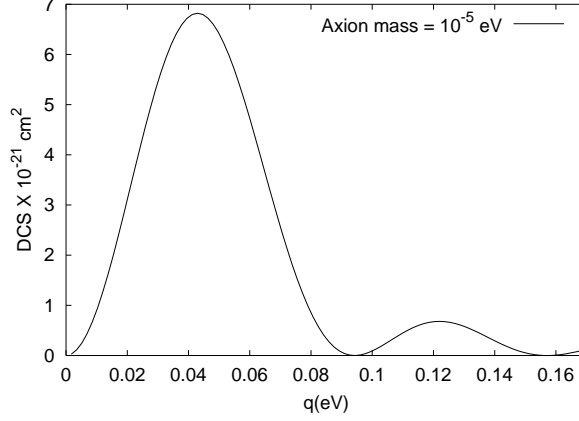


Figure 1: DCS as a function of the photon momentum  $q$ . Axion mass is taken to be  $m_a = 10^{-5}$  eV.

the DCS's ratio  $R$  between the  $TM_{110}$  mode of the resonant cavity and the  $TE_{10}$  mode of the wave guide ( for the best case)

$$R = \frac{(DCS)_{110}}{(DCS)_{10}} \sim \frac{q^2}{\omega^2} \left(1 + \frac{\omega}{q}\right)^2. \quad (12)$$

From Eq.(12) it is easy to see that, in the limit  $\omega^2 = m_a^2 \ll q^2$  then we have  $(DCS)_{110} \gg (DCS)_{10}$ . This means that *the conversion cross sections of the resonant cavity are much larger than those of the wave guide in the same conditions.*

ii) We estimate the result for the best case in C. G. S. Gauss units. Using data as in Refs. [17, 20],  $E_o = 10^6 \text{ cm}^{-1/2} \text{ g}^{1/2} \text{ s}^{-1}$ ,  $a = b = d = 100$  cm and  $\omega = m_a$ , the DCS for Eq.(11) as a function of the momentum of photon was plotted in the figure 1. We can see from the figure, when the momentum of photons is perpendicular to the momentum of axions then there exists *the resonant conversion* at the value  $q \approx 4.7 \times 10^{-2}$  eV, the DCS is given by  $\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} = 6.7 \times 10^{-21} \text{ cm}^2$ . This is due to the limit  $q \rightarrow \frac{\pi}{b}$ . At the high values of the momentum  $q$  then DCS's have very small values.

iii) We emphasize that the explicit resonance happens *only* in the direction *perpendicular* to the direction of the photon while in the static EM fields the axions are produced mainly in the direction of the photon motion.

Finally, it is worth noting that the scattering of photons in an external EM field is the most important effect because it has the following advantages [32]: *Firstly*, this

is the unique effect giving nonzero cross-section in the first order of the perturbation theory. *Secondly*, since the EM field is classical we can increase the scattering probability as much as possible by increasing the intensity of the field or the volume containing the field. *This is a important point in order to apply it in experiments.* However in our work we considered only a theoretical basis for experiments, other problems with axion detection will be investigated in the future.

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